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AN ANALYSIS OF THE EFFECT OF TENSION INDUCED DAMAGE ON CREEP BUCKLING OF COLUMNS

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Abstract—The influence of tension induced Kachanov–Rabotnov type damage on the buckling of a Shanley column model is investigated. Both instantaneous and delayed buckling are shown to occur when the state of the column, represented by load-deflection-damage, reaches a certain surface, the instability surface. The special case of purely brittle buckling is studied in some detail.

1. INTRODUCTION

Tensile members may rupture instantaneously under instantaneously applied load or after a certain time under constant or changing load. The collapse may be caused by one or both of two effects: (a) essential changes in geometry (Poisson effect); (b) creation and growth of damage (here denoted Kachanov effect). Such collapse mechanisms have been examined by Hoff[1], Kachanov[2], Odqvist[3], Broberg[4] and others.

This paper deals with corresponding phenomena for compressive members. A Shanley type column model is analysed, considering both (a) essential changes in geometry and (b) creation and growth of damage. The geometric effect here is the influence of the deflection on the bending moment, which is characteristic of all compressed slender members. In contrast to this the Poisson effect is here of minor importance and will be disregarded.

A previous study, Boström[5], dealt with compressive collapse of a much simplified column model, where all deformation and damage was located in a hinge sensing only the bending moment and not the axial force. That model may not distinguish between tensile and compressive material behaviour and is insensitive to reversals in stress rate, all of which are of importance to the behaviour of a real column, see Fraeijs de Veubeke[6], Hoff[7, 8].

The present study is an extension of the first analysis[5] of damage induced compressive collapse. Here the differences in deformation and damage creation due to various changes in stress and stress rate are taken into account.

2. MODEL STRUCTURE

An obvious improvement over the pure moment hinge model considered in [5] is the two bar hinge model studied by Shanley[9], see Fig. 1. Here different tensile and compressive material behaviour may be taken into account with a minimum of mathematical complication.

Shanley considered an initially straight column subject to an increasing axial load. Here an initially slightly curved column will be analysed in accordance with the pattern set by Hoff[7], who found the initial curvature to be an important parameter in ductile creep buckling.

Experiments, e.g. by Hult[10], have shown that the deflection curve of a metal column subject to creep becomes increasingly more pointed with the lapse of time. This is a consequence of the progressively nonlinear dependence of deformation and damage upon stress. The sharply pointed Shanley column model therefore should be well suited to describe creep buckling in real metal columns.

The Shanley hinge with bars $2d$ apart corresponds directly to an idealized H cross section with flanges $2d$ apart. For other cross sectional forms the distance $2d$ may be chosen to render a good representation of the bending stiffness.

With Q_1 and Q_2 denoting the tensile bar forces equilibrium requires

$$Q_1 = -P(d - \Delta)/2d \tag{1}$$

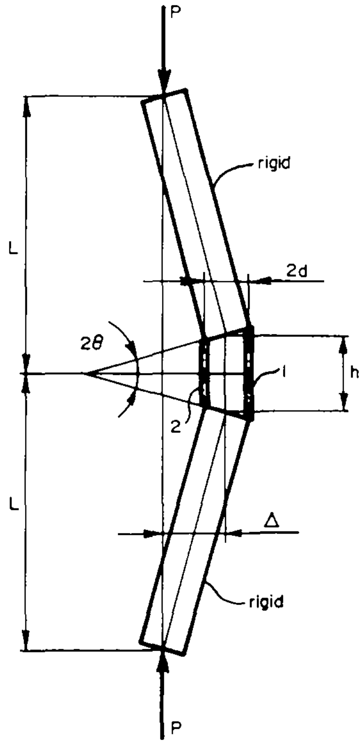

Fig. 1. Column model. All damage and deformation occurs in a hinge, consisting of two bars 1 and 2 with cross sectional area $A/2$ each.

and

$$Q_2 = -P(d+\Delta)/2d. \tag{2}$$

Here P denotes the axial compressive load and Δ the midpoint deflection. The slope Θ has been assumed to be so small that $\cos\Theta \simeq 1$.

With $P > 0$ follows from (2) that $Q_2 < 0$ for all deflections Δ, whereas $Q_1 < 0$ only as long as $\Delta < d$. Hence bar 2 will always be compressed, whereas bar 1 may change from a state of compression to tension during load application or the subsequent creep process.

With ϵ_1 and ϵ_2 denoting the bar strains compatibility requires

$$\epsilon_1 - \epsilon_2 = (2d/hL)(\Delta - \Delta_{00}) \tag{3}$$

where Δ_{00} denotes the initial midpoint deflection, and h is the bar length. The abbreviated notation

$$\lambda = (hL/2d) \tag{4}$$

will be used in the sequel.

3. MODEL MATERIAL

The deformation ϵ and damage ω of the column material, and hence also of the hinge bar elements, are assumed to follow the constitutive relations proposed by Broberg[4]. They were stated as

$$d\epsilon = G'(s)ds + F(s)dt \tag{5}$$

$$d\omega = g'(s)ds + f(s)dt \tag{6}$$

for the case when $s \geq 0$ and $ds \geq 0$. Here the quantity s, denoted net stress, will be defined as

$$s = \sigma/(1-\omega) \tag{7}$$

where σ is the true stress, which, in absence of a Poisson effect, equals the nominal stress.

The relations (5), (6) were applied by Broberg[4] in analysing rupture in bars under increasing or constant tensile loads. For the present problem the constitutive relations have to be extended to describe also cases of negative net stress, which may be decreasing or increasing.

With $A/2$ denoting the cross sectional area of each hinge bar there follows from (1), (2) and (7)

$$s_1 = -(P/A).[(1-\Delta/d)/(1-\omega_1)] \tag{8}$$

and

$$s_2 = -(P/A).[(1+\Delta/d)/(1-\omega_2)]. \tag{9}$$

Comparison with the case of ductile creep buckling ($\omega_1 \equiv \omega_2 \equiv 0$), see Odqvist and Hult[11] p. 262–266, indicates that the relations between P, s_1, s_2 and Δ will be as shown in Figs. 2 and 3. The following assumptions are now made to cover all the constellations of s and $\dot{s}$ occurring ($\dot{s} = ds/dt$).

(1) $s < 0$: Compressive net stress. No damage is created; existing damage is preserved. Hence $g' = 0$, $f = 0$.

(2) $\dot{s} < 0$: Decreasing net stress. No healing occurs. Hence $g' = 0$.

(3) $s.\dot{s} < 0$: Unloading in tensile or compressive regime. Instantaneous strain component is not recovered. Hence $G' = 0$.

(4) $s < 0$, $\dot{s} < 0$: loading in compressive regime. Compressive strain created. Hence $G' < 0$, $F < 0$.

Evidence for assumption (1) is given by Hayhurst[12]. Assumption (2) is not very carefully examined so far. However, since the recoverable part of the strain is here neglected it seems

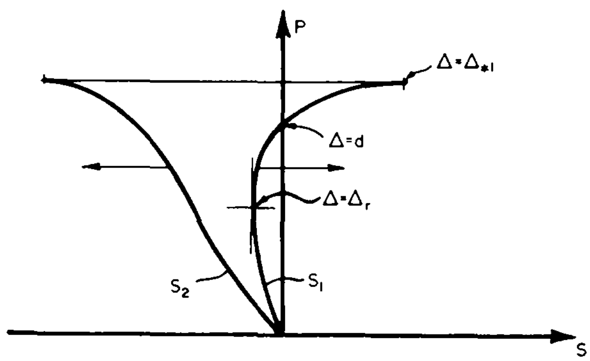

Fig. 2. Load-net stress relations for the case $\Delta_{00} < d$. Horisontal arrows indicate creep at constant P-values. The net stress s_1 needs not be positive when buckling occurs.

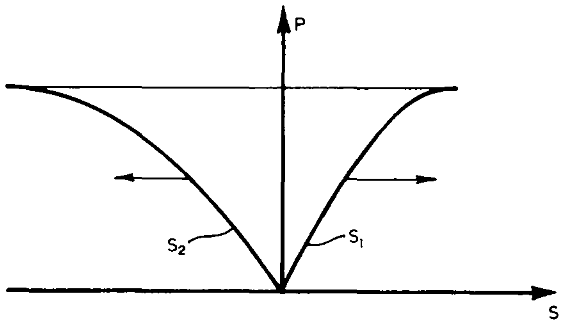

Fig. 3. Load-net stress relations for the case $\Delta_{00} > d$.

reasonable to do so with a possible recoverable part of the damage. If damage is neglected assumptions (3) and (4) are identical to those made by Hoff [7]. The assumptions (1)–(4) lead to the following generalizations of the constitutive relations (5), (6).

$$d\epsilon = G'(|s|)H(s\dot{s})ds + F(|s|)\,\mathrm{sgn}\, s dt \tag{10}$$

$$d\omega = g'(|s|)H(\dot{s})H(s)ds + f(|s|)H(s)dt \tag{11}$$

Here $H(\;)$ denotes the Heaviside unit step function, and sgn( ) the signum function.

In the subsequent analysis the functions G, F, g and f will be taken as simple power functions

$$G(s) = B_0 s^{n_0}, \tag{12}$$

$$F(s) = Bs^n, \tag{13}$$

$$g(s) = C_0 s^{\nu_0}, \tag{14}$$

$$f(s) = Cs^{\nu}. \tag{15}$$

To simplify the mathematics further the exponents n_0 and n will be assumed to be odd integers.

4. GOVERNING EQUATIONS

Considering the net stress histories shown in Figs. 2 and 3, eqns (10)–(15) yield

$$d\epsilon_1 = B_0 n_0 s_1^{n_0-1} ds_1 \,.\, H(s_1\dot{s}_1) + Bs_1^n dt \tag{16}$$

$$d\epsilon_2 = B_0 n_0 s_2^{n_0-1} ds_2 + Bs_2^n dt \tag{17}$$

$$d\omega_1 = C_0 \nu_0 s_1^{\nu_0-1} ds_1 \,.\, H(s_1) + Cs_1^{\nu} dt \,.\, H(s_1) \tag{18}$$

$$\omega_2 = 0 \tag{19}$$

where, according to (8) and (9)

$$s_1 = -(P/A)\,.\,[(1-\Delta/d)/(1-\omega_1)] \tag{20}$$

and

$$s_2 = -(P/A)\,.\,(1+\Delta/d). \tag{21}$$

From (3), (4) and (16)–(21) follow the incremental relations

$$\alpha d\Delta = \beta_0 dP + \beta dt \tag{22}$$

$$\alpha d\omega_1 = \gamma_0 dP + \gamma dt \tag{23}$$

where

$$\alpha = \frac{1}{\lambda}\left\{\left[1 - \frac{PhL}{2I} B_0 n_0 s_2^{n_0-1}\right][1 - \omega_1 - C_0\nu_0 s_1^{\nu_0} H(s_1)] - \frac{PhL}{2I} B_0 n_0 s_1^{n_0-1} H(s_1\dot{s}_1)\right\} \tag{24}$$

$$\beta_0 = \frac{B_0 n_0}{P}\{s_1^{n_0}(1-\omega_1)H(s_1\dot{s}_1) - s_2^{n_0}[1-\omega_1 - C_0\nu_0 s_1^{\nu_0} H(s_1)]\} \tag{25}$$

$$\beta = B_0 C n_0 s_1^{n_0-\nu} H(s_1) + B(s_1^n - s_2^n)[1 - \omega_1 - C_0 \nu_0 s_1^{\nu_0} H(s_1)] \tag{26}$$

$$\gamma_0 = \frac{C_0\nu_0}{P} s_1^{\nu_0} H(s_1) \left\{ \left(1 - \frac{PhL}{2I} B_0 n_0 s_2^{n_0-1}\right) \frac{1-\omega_1}{\lambda} - \frac{B_0 n_0 P s_2^{n_0}}{s_1 A d} \right\} \tag{27}$$

$$\gamma = \frac{H(s_1)}{Ad} \left\{ BC_0\nu_0 s_1^{\nu_0-1} . P(s_1^n - s_2^n) + \frac{2I}{hL} C s_1^{\nu}(1-\omega_1) \right.$$
$$\left. \left(1 - \frac{PhL}{2I} B_0 n_0 s_2^{n_0-1}\right) - B_0 C P n_0 s_1^{\nu+n_0-1} \right\} \tag{28}$$

with

$$I = Ad^2 \tag{29}$$

denoting the area moment of inertia at the hinge.

If s_1 and s_2 according to (20) and (21) are inserted into (24)–(28), the coefficients take the form

$$\alpha = \alpha(P, \Delta, \omega_1), \ldots, \quad \gamma = \gamma(P, \Delta, \omega_1). \tag{30}$$

For any given non decreasing loading history $P = P(t)$ the relations (22), (23) may then be integrated step by step to give $\Delta = \Delta(t)$ and $\omega_1 = \omega_1(t)$, starting from the initial state

$$P = 0, \Delta = \Delta_{00}, \quad \omega_1 = 0. \tag{31}$$

The standard type of loading history in creep buckling studies is step loading

$$P(t) = P_0 H(t). \tag{32}$$

Two phases then appear, viz.

(a) *Load application phase* $(0^- < t < 0^+, 0 \leq P \leq P_0)$

The governing eqns (22), (23) take the form

$$\alpha d\Delta = \beta_0 dP \tag{33}$$

$$\alpha d\omega_1 = \gamma_0 dP \tag{34}$$

where now $\Delta = \Delta(P)$, $\omega_1 = \omega_1(P)$. The starting conditions are $\Delta(0) = \Delta_{00}$, $\omega_1(0) = 0$. If P_0 is not large enough to cause instantaneous buckling, this phase ends in the state $\Delta = \Delta(P_0) \equiv \Delta_0$, $\omega_1 = \omega_1(P_0) \equiv \omega_0$.

(b) *Creep phase*

The governing equations are given by (22), (23) with $dP/dt = 0$. Hence

$$\alpha d\Delta = \beta dt \tag{35}$$

$$\alpha d\omega_1 = \gamma dt. \tag{376}$$

Here $\Delta = \Delta(t)$, $\omega_1 = \omega_1(t)$. The initial conditions are $\Delta(0) = \Delta_0$, $\omega_1(0) = \omega_0$ as caused by the load application.

5. INSTABILITY CONDITIONS

The column may become unstable in the sense that the deflection Δ and damage ω_1 increase at an unlimited rate. This may occur during the load application phase and then implies that

$$d\Delta/dP \to \infty, \quad d\omega_1/dP \to \infty \tag{37}$$

or during the creep phase and then implies that

$$d\Delta/dt \to \infty, \quad d\omega_1/dt \to \infty. \tag{38}$$

The conditions for these two kinds of instability are given by the governing eqns (33), (34) and (35), (36) respectively. One common condition is found for *instantaneous* instability (37) and *delayed* instability (38), viz.

$$\alpha = 0 \tag{39}$$

where α is given by (24). Since $\alpha = \alpha(P, \Delta, \omega_1)$, eqn (39) defines an instability surface in $P - \Delta - \omega_1$-space, see Fig. 4. This surface is independent of the loading history as long as the load is a non decreasing function of time.

The changing state of the column is described by a path in this space, starting at point A. Load application corresponds to the path A–B–C, where C is the instability point located on the instability surface. The load P_{*i}, is termed the instantaneous instability load (buckling load).

If $P_0 < P_{*i}$, a creep and damage process will succeed the loading process, and a path BD will be followed. This path terminates in the instability point D, corresponding to delayed instability. The finite time required for the point D to be reached is called the creep buckling time.

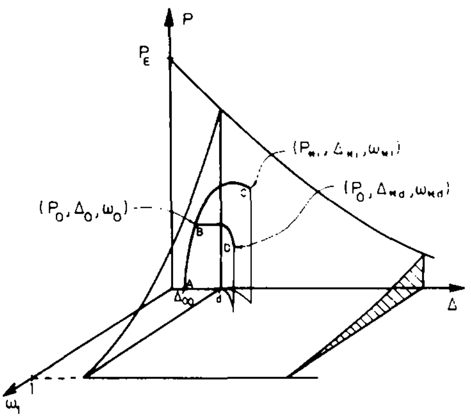

Fig. 4. Instability surface in P-Δ-ω_1-space. One case of instantaneous buckling ABC and one case of creep buckling ABD shown. For these cases $\omega_0 = 0$ since $\Delta_0 < d$.

6. STRESS RATE AND STRESS REVERSALS

Determination of the path ABD requires reversals in stress rate and stress to be considered, as indicated by the functions $H(s_1)$ and $H(s_1\dot{s}_1)$ appearing in (24)–(28). Different possible situations appear from Figs. 2 and 3. Fig. 3 shows the behaviour when $\Delta_{00} > d$. Obviously $H(s_1) = 1$ and $H(s_1\dot{s}_1) = 1$ throughout a process starting at $\Delta_{00} > d$. If, however, $\Delta_{00} < d$ the load application process can be divided into three parts referring to Fig. 2, viz.

(1) $\Delta_{00} < \Delta < \Delta_r$; $H(s_1) = 0$, $H(s_1\dot{s}_1) = 1$. Here Δ_r is the deflection at the moment of stress rate reversal. (2) $\Delta_r < \Delta < d$; $H(s_1) = 0$, $H(s_1\dot{s}_1) = 0$. Stress reversal occurs when $\Delta = d$. (3) $\Delta > d$: $H(s_1) = 1$, $H(s_1\dot{s}_1) = 1$.

Now assume that the load is held constant $P = P_0$ after load application. Then $\dot{s}_1 > 0$ for the complete creep process starting at $\Delta = \Delta_0$. Hence stress rate reversal occurs at $\Delta = \Delta_r$ if $\Delta_0 > \Delta_r$ else it occurs at $\Delta = \Delta_0$. Stress reversal takes place when $\Delta = d$.

Equations (1)–(4) and (16)–(19) give the deflection at stress rate reversal as the solution of

$$(d - \Delta_r)[(d + \Delta_r)^{n_0} - (d - \Delta_r)^{n_0}] = 2n_0 d(d + \Delta_r)^{n_0 - 1}(\Delta_r - \Delta_{00}). \tag{40}$$

The net stress in flange 1 then is

$$s_{1r} = -(d - \Delta_r)\left[\frac{1}{B_0\lambda} \cdot \frac{\Delta_r - \Delta_{00}}{(d + \Delta_r)^{n_0} - (d - \Delta_r)^{n_0}}\right]^{1/n_0}. \tag{41}$$

7. COMPLETELY DUCTILE BEHAVIOUR

If damage does not occur, $C_0 = C = 0$ in (14)–(15), the buckling process is completely ductile. The net stresses s are identical to the nominal stresses σ. A detailed analysis of this case is given by Hoff [7]. He examined the buckling of an idealized H cross section column. His constitutive equations were as given by (9) if s is changed to σ and a term corresponding to elastic deformation is added. The results are quoted below in terms of the basic equations and the notation used here.

I. The load deflection relation for instantaneous loading is

$$P = \frac{Ad}{(B_0\lambda)^{1/n_0}}\left\{\frac{\Delta - \Delta_{00} - B_0\lambda s_{1r}^{n_0} H(\dot{s}_1)}{(\Delta - d)^{n_0} H(s_1\dot{s}_1) + (\Delta + d)^{n_0}}\right\}^{1/n_0} \tag{42}$$

where

$$H(\dot{s}_1) = \begin{cases} 0 & \text{when } \Delta < \Delta_r \\ 1 & \Delta_r < \Delta \end{cases}$$

$$H(s_1\dot{s}_1) = \begin{cases} 1 & \text{when } \Delta_{00} < \Delta < \Delta_r. \\ 0 & \Delta_r < \Delta < d \\ 1 & d < \Delta. \end{cases}$$

II. The time-deflection relation for creep at $P = P_0$ starting with $\Delta = \Delta_0$ when $t = 0$ is given by

$$t = \frac{n_0 B_0}{B}\left(\frac{P}{Ad}\right)^{n_0 - n}\int_{\Delta_0}^{\Delta}\frac{\frac{1}{n_0 B_0\lambda}\left(\frac{Ad}{P_0}\right)^{n_0} - \{(\Delta - d)^{n_0-1} H(s_1\dot{s}_1) - (\Delta + d)^{n_0-1}\}}{(\Delta - d)^n + (\Delta + d)^n}\, d\Delta. \tag{43}$$

III. The instability condition (39) is given by inserting $\omega_1 = C_0 = 0$ in (24). The condition common for instantaneous and delayed instability is

$$n_0 B_0 \lambda\left(\frac{P_*}{Ad}\right)^{n_0}\{(\Delta_* + d)^{n_0-1} + (\Delta_* - d)^{n_0-1} H(s_{1*}\dot{s}_{1*})\} = 1. \tag{44}$$

Index $*$ here and in the sequel stands for conditions at instability.

The intersection between (42) and (44) gives the load P_{*i} and deflection Δ_{*i} at instantaneous instability for a certain value of Δ_{00}. Instantaneous instability cannot occur when $\Delta < \Delta_r$ i.e. stress rate reversal must take place before buckling.

The intersection between (44) and the P axis in Fig. 4 corresponds to a bifurcation load

$$P_E = A\left\{\frac{d}{2\lambda B_0 n_0}\right\}^{1/n_0} \tag{45}$$

for an initially perfect column. In that case $\Delta_{*i} = \Delta_r = \Delta_{00} = 0$.

The time t_* consumed at delayed instability is given by (43) where the upper limit of Δ is taken as the solution Δ_{*d} of (44) under the condition $P = P_0$. The lower limit is calculated from (42) with $P = P_0$.

8. COMPLETELY BRITTLE BEHAVIOUR

This case as well as the previous one is a limiting case in the present context. It concerns hypothetical nondeforming materials. Therefore eqns (3), (16) and (17) are replaced by the condition $\Delta = \Delta_{00}$. Then if $\Delta_{00} > d$, s_1 will be positive and hence eqn (18) yields nonzero damage. Else if $\Delta_{00} < d$ damage will not occur. In the sequel of this section it is assumed that $\Delta_{00} > d$.

(a) *Load application.* $0 < P < P_0$

Equations (8) and (18) give the load-damage relation for instantaneous loading

$$P = \frac{Ad}{C_0^{1/\nu_0}(\Delta_{00} - d)}\,\omega_1^{1/\nu_0}\,.\,(1 - \omega_1). \tag{46}$$

Instantaneous instability defined by the condition $d\omega_1/dP = \infty$ occurs when (39) is satisfied. Inserting $B_0 = 0$ and $H(s_1) = 1$ in (39), (24) and using (8) yields the condition common for instantaneous and delayed instability

$$P_* = (1 - \omega_*)^{(\nu_0+1)/\nu_0}(C_0\nu_0)^{-1/\nu_0}\cdot\frac{Ad}{\Delta_{00} - d}. \tag{47}$$

The load according to (46) satisfies the instability condition, when the damage is

$$\omega_{*i} = 1/(1 + \nu_0). \tag{48}$$

Inserting $\omega = \omega_{*i}$ in (46) gives the load at instantaneous collapse

$$P_* = \left(\frac{\nu_0}{1 + \nu_0}\right)^{(1+\nu_0)/\nu_0}(\nu_0 C_0)^{-1/\nu_0}\frac{Ad}{\Delta_{00} - d}. \tag{49}$$

According to eqn (48) it is obvious that damage at instantaneous brittle instability is independent of Δ_{00} as long as $\Delta_{00} > d$.

(b) *Constant load* $P = P_0$

If $P_0 < P_*$ collapse does not occur during load application but after a certain time t_*. The damage ω_0 at time $t = 0$ is given by (46) if $P = P_0$ is inserted

$$Ad(1 - \omega_0)\omega_0^{1/\nu_0} = P_0 C_0^{1/\nu_0}(\Delta_{00} - d). \tag{50}$$

Delayed instability occurs when $\omega_1 = \omega_{*d}$, which according to the instability condition (47) is

$$\omega_{*d} = 1 - (C_0\nu_0)^{1/(1+\nu_0)}\,.\,k^{\nu_0/(1+\nu_0)} \tag{51}$$

where

$$k = (P_0/A)\{(\Delta_{00} - d)/d\}. \tag{52}$$

The time t_* at which the column ceases to be stable under the load P_0 can be calculated from eqn (36). Taking α and γ from (24) and (28) and putting $B = B_0 = 0$ and $\Delta = \Delta_{00}$ yields upon integration the time-damage relation

$$t = \frac{C_0\nu_0 k^{\nu_0-\nu}}{C(\nu - \nu_0)}[(1 - \omega_1)^{\nu-\nu_0} - (1 - \omega_0)^{\nu-\nu_0}] - \frac{(1 - \omega_1)^{\nu+1} - (1 - \omega_0)^{\nu+1}}{Ck^{\nu}(\nu + 1)}. \tag{53}$$

Inserting $\omega_1 = \omega_{*d}$ according to (51) gives the time at delayed instability

$$Ck^{\nu}t_* = \frac{1 + \nu_0}{(1 + \nu)(\nu - \nu_0)}(\nu_0 C_0 k^{\nu_0})^{(1+\nu)/(1+\nu_0)} + \frac{(1 - \omega_0)^{1+\nu}}{1 + \nu} - \frac{\nu_0 C_0 k^{\nu_0}}{\nu - \nu_0}(1 - \omega_0)^{\nu-\nu_0}. \tag{54}$$

For materials which exhibit very small initial damage there follows by putting $C_0 = \omega_0 = 0$ in (54)

$$t_* = \frac{1}{C(1 + \nu)}(A/P_0)^{\nu}\,.\,[d/(\Delta_{00} - d)]^{\nu}. \tag{55}$$

9. DISCUSSION

The influence of material damage on buckling at elevated temperature has been studied. This subject was dealt with in a previous paper (Boström [5]), where a simple hinge model column was examined. A characteristic of this model is that for every initial deflection $\Delta_{00} > 0$ there exists a finite collapse load P_*. Moreover for creep under a load $P_0 < P_*$ there exists a finite creep buckling time t_*. This is true also for the limiting case of purely brittle behaviour (only damage, no deformation). A different behaviour is found for the Shanley model studied in the present paper. In the limiting case of purely brittle behaviour neither instantaneous nor delayed collapse will occur if $\Delta_{00} < d$. Such a column will be safe at any load and any time if only it is straight enough.

Both the simple hinge model and the Shanley model in combination with the present constitutive equations become unstable when the state (P, Δ, ω_1) reaches a certain instability surface $\alpha(P_*, \Delta_*, \omega_*) = 0$. For the simple hinge model the instability condition has no point common with the P-axis. The initially straight column is then stable for all values of P and t. In the present study the instability condition is a surface for $\Delta > \Delta_{00}$ and a curve in the P-Δ-plane for $\Delta < \Delta_{00}$. This curve intersects the P-axis at a certain load value P_E given by eqn (45). A finite buckling load exists even for the initially straight column. If, however, the load is held constant in time at a value less than that bifurcation load the perfectly straight column will not loose its stability at any time. To achieve a buckling time for the perfectly straight column another approach has to be made. Such analysis has been performed by Rabotnov and Shesterikov [13].

For mixed ductile and brittle behaviour every initial imperfection will lead to buckling, however small a load is applied. Most materials used for structural members under creep conditions exhibit small rupture deformations when subjected to low stresses. The material behaviour becomes more brittle at low stress levels. For the model studied here that corresponds to large values of $d\omega/d\Delta$ for low values of P_0. The warning in form of deformations prior to buckling will be less pronounced when the load P_0 is small.

REFERENCES

1. N. J. Hoff, Necking and rupture of rods under tensile loads. *J. Appl. Mech.* **20**, 105 (1953).
2. L. M. Kachanov, On the time to failure under creep conditions (in Russian). *Izv. Akad. Nauk. SSSR* **8** (1958).
3. F. K. G. Odqvist, *Mathematical Theory of Creep and Creep Rupture.* Oxford University Press, London (1974).
4. H. Broberg, A new criterion for brittle creep rupture. *J. Appl. Mech.* **41**, 809 (1974).
5. P. O. Boström, Creep buckling considering material damage. *Int. J. Solids Structures* **11**, 765 (1975).
6. B. Fraeijs de Veubeke, Creep buckling. *High Temperature Effects In Aircraft Structures* (Edited by N. J. Hoff), Chap. 13. Pergamon Press, London (1958).
7. N. J. Hoff, Creep buckling. *PIBAL Report No.* 252, Polytechnic Institute of Brooklyn, Brooklyn, N.Y. May 1954; also *Aeronaut. Quart.* 7, 1 (1956).
8. N. J. Hoff, Creep buckling of plates and shells. *Proc.* XIII *Int. congr. Theor. and Appl. Mech.* Moscow 1972, (Edited by E. Becker and G. K. Mikhailov). Springer, Berlin (1973).
9. F. R. Shanley, Inelastic column theory. *J. Aeronaut. Sciences* **14**, 261 (1947).
10. J. A. H. Hult, Creep buckling, KTH, Stockholm, Inst. f. Hållfasthetslära, Publ. No. 111 (1955).
11. F. K. G. Odqvist and J. A. H. Hult, *Kriechfestigkeit Metallischer Werkstoffe.* Springer, Berlin (1962).
12. D. R. Hayhurst, Creep rupture under multi-axial states of stress. *J. Mech. Phys. Solids* **20**, 381 (1972).
13. Yu. N. Rabotnov and S. A. Shesterikov, Creep stability of columns and plates. *J. Mech. Phys. Solids* **6**, 27 (1957).